Indian J. Phys. **73A** (1), 123–131 (1999)

IJP A
— an international journal

Supersymmetry breaking and CP violation in orbifold compactified string theories

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Abstract : Compactification of the six unobserved spatial dimensions of string theory onto an orbifold generates loop corrections to the gauge kinetic function in the effective supergravity theory which depend upon the orbifold moduli. The perturbative potential for the moduli is flat, but the corrections to the gauge kinetic function can generate a non-perturbative superpotential for the hidden sector gaugino condensate, and hence a non-perturbative potential for the moduli. The minimum of this potential fixes the moduli expectation values, and hence determines the soft supersymmetry-breaking masses, and scalar bilinear (B) and trilinear (A) terms in the perturbative potential. Generically, the moduli have complex expectation values, so the soft supersymmetry-breaking induces CP violation. We discuss the consistency of the predicted CP violation with bounds deriving from experiments searching for an electric dipole moment of the neutron.

Keywords : Supersymmetry, CP violation, string theory

PACS Nos. : 11.15.Ex, 11.30.Pb, 11.25 -w, 11.30.Er

1. Modular symmetries and moduli

If a single string world sheet coordinate X (τ, σ) is compactified on a circle of radius R, so that

$$X(\tau, \sigma+\pi) = X(\tau, \sigma) + 2\pi Rn,$$

then the zero-mode part of the world sheet becomes

$$X^{(0)}(\tau, \sigma) = x + \frac{m}{R}\tau + 2Rn\sigma$$

*All of the work reported was done in collaboration with Alex Love and parts of it with Bobby Acharya, Wafic Sabra, Steven Thomas and George Kraniotis.
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with m and n integers. The integer n labels the winding number and m is the momentum quantum number, arising as usual on a lattice. Then the Virasoro operators L_0 and $\bar{L}_0$ are both invariant under the combined transformations

$$m \leftrightarrow n \quad \sqrt{2}R \leftrightarrow \frac{1}{\sqrt{2}R}.$$

This is the simplest example of the (large-small) duality symmetry which underlies the string loop threshold corrections to the running of the gauge coupling constants in string theory. The symmetry is enlarged when the compactification is on a torus, for example, on a six-torus

$$T^6 = T^2 \times T^2 \times T^2.$$

A modular symmetry is defined as an integral matrix transformation of the momentum and winding quantum numbers $\begin{pmatrix} m^a \\ n^a \end{pmatrix}(a = 1, \cdots, 6)$ leaving L_0 and $\bar{L}_0$ invariant. In each torus T^2 we have a metric $G_{ij}(i, j = 1, 2)$ and antisymmetric field B_{12} and we define the *moduli*

$$U = \frac{1}{G_{11}}\left(\sqrt{\det G} - iG_{12}\right),$$

$$T = 2\left(\sqrt{\det G} - iB_{12}\right),$$

so that U encodes the complex structure and T the overall scale. Then the modular symmetry group is three copies, one for each plane, of

$$PSL(2, Z)_T \times PSL(2, Z)_U,$$

where $PSL\,(2, Z)_T$ is the group of transformations

$$T \to \frac{aT + ib}{icT + d}$$

with $\qquad a, b, c, d \in Z \quad \text{and} \quad ad - bc = 1$

and similarly for the U modulus. The group is generated by the two transformations

$$T \to T + \mathrm{i},$$

$$T \to \frac{1}{T}.$$

Thus we define a fundamental domain $\mathcal{F}$

$$\mathcal{F} = \left\{T \middle| -1/2 < \operatorname{Im} T < 1/2,\ |T| \geqslant 1\right\}$$

from which all values of T can be reached by modular transformations. For future reference we note that the points $T = 1$ and $T = e^{i\pi/6}$ in $\mathcal{F}$ are fixed points under the transformation $T \to 1/T$. However, toroidal compactifications give $N = 4$ space-time supersymmetries, whereas for phenomenolgical reasons we require $N = 1$ and it is for this reason that orbifold compactifications are considered.

2. Orbifolds

An orbifold Ω is obtained from a torus by identifying points which are related under the action of a discrete "point group" P

$$\Omega = T^6/P,$$

where the elements θ of P act on the points x of T^6 by

$$\theta : x \rightarrow \theta x + l,$$

where l is a vector belonging to the lattice Λ defining the torus. Of course P must act crystallographically on Λ, so that for each basis vector e^a of Λ

$$\theta e^a = Q^a_b e^b \quad \text{with} \quad Q^a_b \in Z$$

and the requirement of $N = 1$ supersymmetry restricts the possible (abelian) point groups to various Z_N and $Z_M \times Z_N$ groups [1]. The winding and momentum quantum numbers transform under the action of P as

$$n \rightarrow Qn \quad \text{and} \quad m \rightarrow Q^{t^{-1}} m$$

and the requirement that L_0 and $\bar{L}_0$ are invariant gives

$$Q^t (G \pm B) Q = G \pm B,$$

where now G, B relate to the whole six-dimensional compactified space. Then, unless θ acts (in a particular plane) as a Z_2 point group, the U modulus is fixed. Thus an orbifold compactification generically has *less* modular symmetry than a toroidal one, with typically a $PSL(2, Z)$ symmetry for each unfixed modulus.

If a Z_N orbifold is generated by a point group element θ, then in the θ^n twisted sector of the orbifold the boundary conditions are

$$X(\tau, \sigma + \pi) = \theta^n X(\tau, \sigma) + 1$$

and these fix the momentum components to be zero in all planes where θ^n acts non-trivially; there is no modular symmetry associated with such a plane. For example the $Z_6 - II$ orbifold is generated by a point group element θ which has eigenvalues

$$\exp \frac{2\pi i}{6} (2, 1, -3).$$

Clearly θ^n acts non-trivially in the second complex plane in which θ is diagonalized for all (non-zero) values of n. However, in the θ^2 twisted sector the point group acts as the identity in the third of the complex planes and in the θ^3 twisted sector it does so in the first complex plane. Further, θ acts as a Z_2 point group in the third plane, so the U modulus associated with this plane is unfixed. The Z_6 point group can be realized as a symmetry by taking the generator θ to be the Coxeter element on an $SO(8) \times SU(3)$ lattice and in this case the modular symmetry is

$$PSL(2,Z)_{T_1} \times PSL(2,Z)_{T_3} \times PSL(2,Z)_{U_3}\cdot$$

A novel situation arises, however, if we realize the Z_6 on a lattice which does not decompose into the sum of a four-dimensional and a two-dimensional lattice. In this case only congruent subgroups of the $PSL\ (2, Z)$ symmetries survive. For example, if the Z_6 is realized as the Coxeter element of an $SU(6) \times SU(2)$, the modular symmetry is reduced to

$$PSL(2,Z)_{T_1} \times \Gamma^0(3)_{T_3} \times \Gamma^0(3)_{U_3},$$

where $\Gamma^0(3)_T$ is defined as the subgroup of $PSL\ (2, Z)_T$ with elements $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ having $b = 0 \bmod 3$.

3. Threshold corrections and supersymmetry breaking

The running of the gauge coupling constants is affected by the existence of string loop threshold corrections and these corrections depend upon the $N = 2$ moduli [2], *i.e.* those associated with planes in which a twisted sector point group element acts trivially. The gauge coupling constant $\alpha_a(\mu)$ at renormalization scale μ is related to that at scale M by

$$\frac{1}{\alpha_a(\mu)} - \frac{1}{\alpha_a(M)} + \frac{b_a}{2\pi}\ln\frac{M}{\mu} + \frac{1}{4\pi}\Delta_a,$$

where b_a is the standard beta function contribution, and the threshold corrections Δ_a depend upon the moduli and possess the relevant modular symmetry. In general, the corrections have the form

$$\Delta_a = -\sum \frac{b_a^i |P^i|}{|P|}\frac{c_{im}}{2}\ln\left[\left(T_i + \bar{T}_i\right)\left|\eta(T_i/l_{im})\right|^2\right],$$

where η is the Dedekind function and c_{im} and l_{im} are integers which differ from unity for the orbifolds with lattices which do not decompose into $T^2 + T^4$. The holomorphic part of Δ_a determines the correction to the gauge kinetic function f_a in the associated effective supergravity theory

$$f_a = S - \frac{1}{8\pi^2}\sum \frac{b_a^i |P^i|}{|P|} c_{im} \ln\left[\eta(T_i/l_{im})\right]$$

where S is the dilaton. Then the gauge term in the effective Lagrangian is

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4} f_a W_\alpha^a W^{a\alpha}\Big|_F + h.c.,$$

where W_α^a is the (spinor-valued) field strength chiral superfield for the gauge group G_a. In supergravity theory under a supersymmetry transformation the change in the fermionic field ψ_i in a chiral supermultiplet with scalar component ϕ_i is

$$\delta\psi_i = \cdots \sum \frac{\partial f_a}{\partial\varphi^j} G_i^{-1j} \lambda_a \lambda_a \xi$$

where ξ is the Grassmann variable parametrizing the transformation, and λ_a is the gaugino field. The quantity $G = K + \ln|W|^2$, where K is the Kähler potential and W the superpotential, determines the matrix

$$G_i^j \equiv \frac{\partial^2 G}{\partial\varphi^i \partial\varphi^{*j}}$$

and G^{-1} is its inverse. Thus, if

$$\frac{\partial f_a}{\partial\varphi^j} \neq 0,$$

we get supersymmetry breaking ($\delta\psi_i \neq 0$) provided there is gaugino condensation, *i.e.* $\langle\lambda_a\lambda_a\rangle_0 \neq 0$.

To find out if gaugino condensation occurs [3] we need a (non-perturbative) potential for the quantity $<\lambda_a\lambda_a>$. For a single gauge group G, we define the composite chiral superfield

$$U = W_\alpha W^\alpha.$$

Then the form given above for $\mathcal{L}_{\text{gauge}}$ requires that the non-perturbative superpotential

$$W^{np} \supset \frac{1}{4} fU,$$

where, as we have seen, the gauge kinetic function f depends upon the (unfixed) moduli. However, the requirement that W^{np} produces the correct chiral and conformal anomalies necessitates an additional term [2]

$$W^{np} = \frac{1}{4}U\left(f - \frac{b}{24\pi^2}\ln U\right),$$

where b is the leading beta function coefficient in the gauge coupling constant renormalization group. Further, we require that W^{np} has modular weight -1 with respect to each modulus, so that under the transformation of the modulus T

$$W^{np} \to (icT + d)^{-1} W^{np}.$$

This in general, requires additional T-dependence to be imported into W^{np} to compensate for the non-invariance of f. It is a good approximation to eliminate U by using

$$\frac{\partial W^{np}}{\partial U} \simeq 0$$

and then the superpotential is a function of the moduli alone

$$W^{np} = \frac{\Omega(\Sigma)}{\prod_{i,m} \eta(T_i / l_{im})^{c_{im}}},$$

where
$$\Sigma \equiv S + \sum_{i,m} \frac{\delta^i_{GS} C_{im}}{8\pi^2} \ln \eta(T_i / l_{im}),$$

$$\Omega(\Sigma) = \sum_a d_a \exp\left(\frac{24\pi^2}{b_a}\right),$$

d_a are (unknown) constants, and δ^i_{GS} are the Green-Schwarz anomaly cancelling coefficients. Finally, using the tree level form for the Kähler potential

$$K = -\ln Y - \sum_i \ln(T_i + \bar{T}_i),$$

where
$$Y = \Sigma + \bar{\Sigma} - \sum_{i,m} \frac{\delta^i_{GS}}{8\pi^2} \ln\left[(T_i + \bar{T}_i)\left|\eta(T_i / l_{im})\right|^2\right]$$

we can calculate the non-perturbative effective potential [3]

$$V^{np}_{\text{eff}} = Y^{-1} \prod_i (T_i + \bar{T}_i)^{-1} \left\{ |W - YW_S|^2 - 3|W|^2 \right.$$

$$\left. + \sum_i \frac{Y}{Y - \frac{\delta^i_{GS}}{8\pi^2}} \left| W - \frac{\delta^i_{GS}}{8\pi^2} W_S - (T_i + \bar{T}_i) W_{T_i} \right|^2 \right\},$$

where $W_S \equiv \partial W / \partial S$ *etc*. In principle, the minimum determines the values of the moduli $\langle S \rangle_0$ and $\langle T_i \rangle_0$. Including the perturbative contributions to W

$$W^p = \sum \lambda_{\alpha\beta\gamma} \varphi_\alpha \varphi_\beta \varphi_\gamma + \sum \mu_{\alpha\beta} \varphi_\alpha \varphi_\beta + \cdots,$$

we can then calculate the soft supersymmetry-breaking gaugino mass terms, scalar masses, as well as the scalar trilinear (A) and bilinear (B) terms [4]

$$V \supset \sum A_{\alpha\beta\gamma} \lambda_{\alpha\beta\gamma} \varphi_\alpha \varphi_\beta \varphi_\gamma + \sum B_{\alpha\beta} \mu_{\alpha\beta} \varphi_\alpha \varphi_\beta.$$

4. CP violation results

If the minimum of $V^{np}_{\rm eff}$ occurs when the moduli are complex, the A and B terms may acquire CP-violating phases [5]. They depend upon the moduli *via* the modified Eisenstein function

$$\hat{G}^i \equiv (T_i + \overline{T}_i)^{-1} + \sum_{m} c_{im} \frac{\partial}{\partial T_i} \ln \eta(T_i / l_{im})$$

and if this is complex at the minimum of $V^{np}_{\rm eff}$, then in general so are $A_{\alpha\beta\gamma}$ and $B_{\alpha\beta}$. The phases $\phi(A)$, $\phi(B)$ of these terms are constrained by experiments searching for a non-zero electric dipole moment of the neutron. These yield [5]

$$\phi(A), \phi(B) \lesssim 5 \times 10^{-3},$$

so the question is whether these bounds are naturally satisfied in string theory, or whether they can be used to exclude some or all orbifold compactifications.

Since there is no *a priori* reason for $<T_i>_0$ to be real, one's expectation is that the latter situation is more likely to arise. However, it must be borne in mind that if $<T_i>_0$ is at a fixed point of the modular transformation, then $\hat{G}^i = 0$, and the anticipated CP-violation does not occur. Also, if $<T_i>_0$ is near to a fixed point, then the CP-violation is small. Further, whenever the real part of $<T_i>_0$ is larger than the fixed-point value, the phase of $\hat{G}^i$, and hence the CP violation, is again small. Finally, the minimum of $V^{np}_{\rm eff}$ always occurs at a fixed point of $\hat{G}^i$ if the Green-Schwarz parameter δ^i_{GS} is zero, so there is no CP violation in this case either. δ^i_{GS} enters in the combination $\delta^i_{GS}/8\pi^2$, which is of order 10^{-1} in general, so this scales all CP violation. For all of these reasons the conclusion on the likely scale of orbifold CP violation is far from foregone.

In any case, we cannot carry out the envisaged programme in the case of a single gaugino condensate, since the value of the dilaton expectation value at the minimum cannot be near the value required by the "observed" unification of the gauge coupling strengths, which corresponds to

$$Y \sim 4.$$

Thus, we are forced to use more than one condensate and the function $\Omega(\Sigma)$ is unknown. We therefore take the quantity $\rho \equiv \Omega_\Sigma/\Omega$ to be a (real) parameter, as was done in [6], and determine the dependence of the values of the unfixed moduli T_i, U_i at the minimum of $V^{np}_{\rm eff}$ on the parameter ρ. Early calculations for the non-decomposable Z_6 – II orbifold indicated that milliweak CP violation occurs naturally in orbifold models [7]. In fact, it appeared that the bounds deriving from the neutron electric dipole moment search could be violated by B-terms for large values of ρ.

However, more recent, higher precision calculations indicate that this is *not* the case, and that the CP-violating phases are very small, of order 10^{-4}. In any case, the early calculations relate to an unrealistic scenario : the value of V_{eff}^{np} at the minimum is negative, corresponding to a negative cosmological constant Λ. One is tempted to take the position that cosmological considerations are beyond the scope of particle theory, and to regard the problem as someone else's. However, the negative value of Λ leads immediately to negative scalar masses, which signal string scale spontaneous gauge symmetry breaking, so the problem cannot be ignored. We therefore arrange [8] that $\Lambda = 0$ by including further, (modular invariant) dilaton-like contributions (X) to the Kähler potential, with no interactions with any other fields. We assume that the X contributions are non-zero and precisely cancel the other contributions at the minimum, thereby giving zero cosmological constant. This undoubtedly *ad hoc* procedure at least has the virtue that it preserves the modular invariance. When we repeat our calculations in this case, the values of the moduli which minimize V_{eff}^{np} are very close to modular group fixed point values, even for very large values of δ_{GS}^{i}, and the predicted CP-violation is negligible, of order 10^{-7}.

The earlier work is also deficient in that it used the tree-level form for the Kähler potential, and there is some evidence that loop corrections may be large [9]. This criticism may be met by taking the Kähler potential to be an arbitrary function $K(Y)$ of Y. We then repeat our calculations treating the derivatives K' and K'' as parameters, and investigate the dependence of the values of the unfixed moduli at the minimum of on these parameters. We find that the moduli can be moved well away from the fixed point values, at any rate for $K'' > K'$. At the time of speaking, we are finding that the imaginary parts

$$\mathrm{Im}\ \hat{G}^{i} \lesssim 10^{-4}$$

and the calculations of the CP-violating phases are in progress. It remains to be seen whether these will constrain the Kahler potential.

Acknowledgments

It is a great pleasure to speak at a particle physics meeting in the city which I first visited thirty years ago, and in which I am now the proud owner of a beautiful house. I thank the Organizing Committee, especially Professor Dilip Choudhury, for giving me the opportunity to do so.

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